Elementary Quantum Mechanics
F. Y. Sem - II

Prof. J. K. Baria
Professor of Physics
V P & R P T P Science College,
Vallabh Vidyanagar 388 120

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Basic Laws of Radiation

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➡ This is the Stefan Boltzmann Law

$$\mathbf{F = \sigma T^4}$$

F = flux of energy (W/m^2)
T = temperature (K)
$\sigma = 5.67 \times 10^{-8}\ W/m^2K^4$ (a constant)

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$$\lambda_{max} \cong \frac{3000\ \mu m}{T(K)}$$

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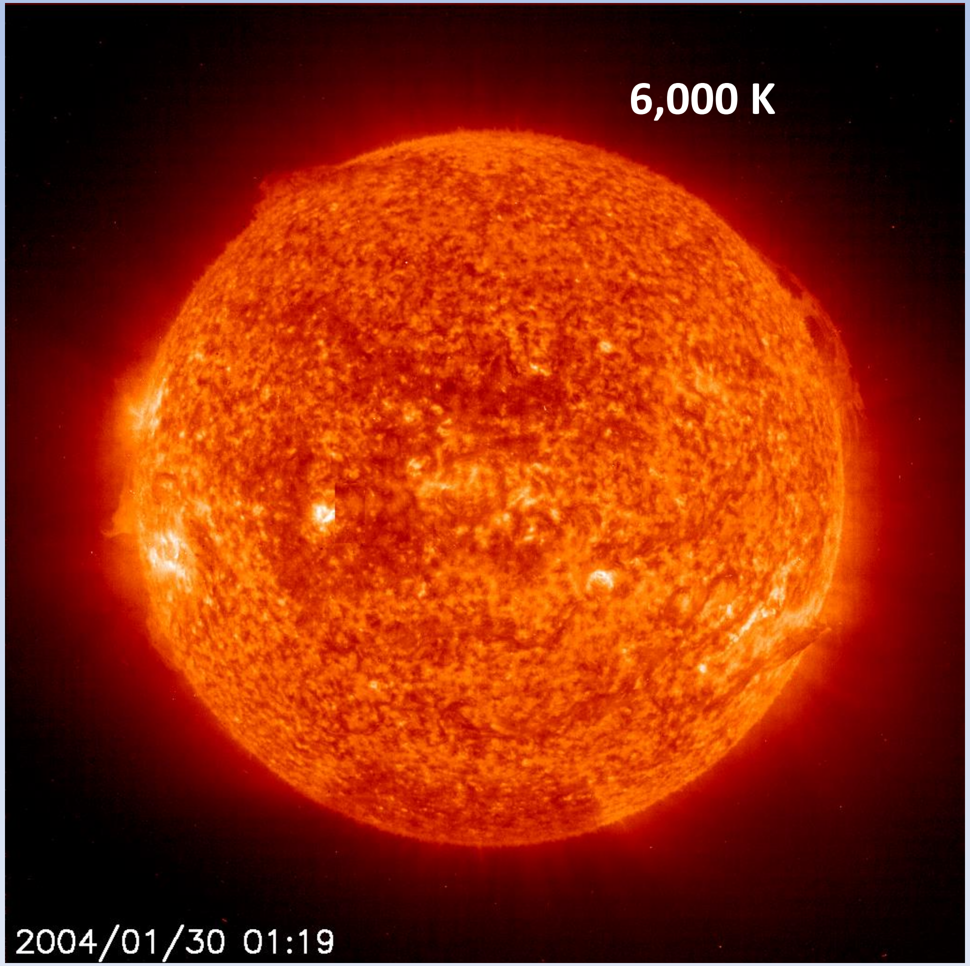
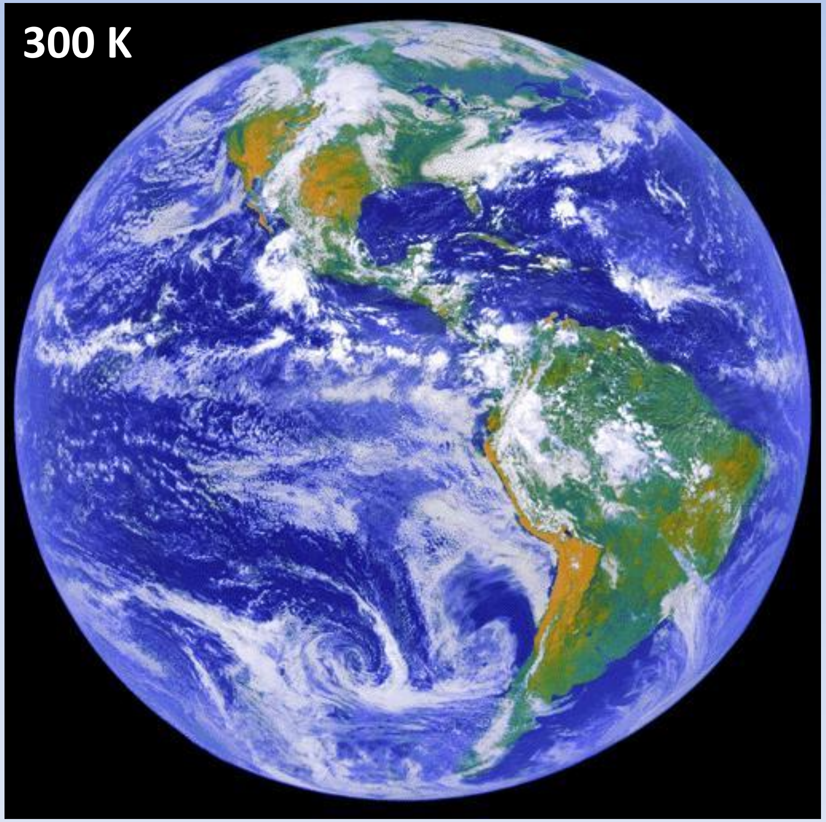
We can use these equations to calculate properties of energy radiating from the Sun and the Earth.





	T (K)	λ_{max} (μm)	region in spectrum	F (W/m^2)
Sun	6000			
Earth	300			

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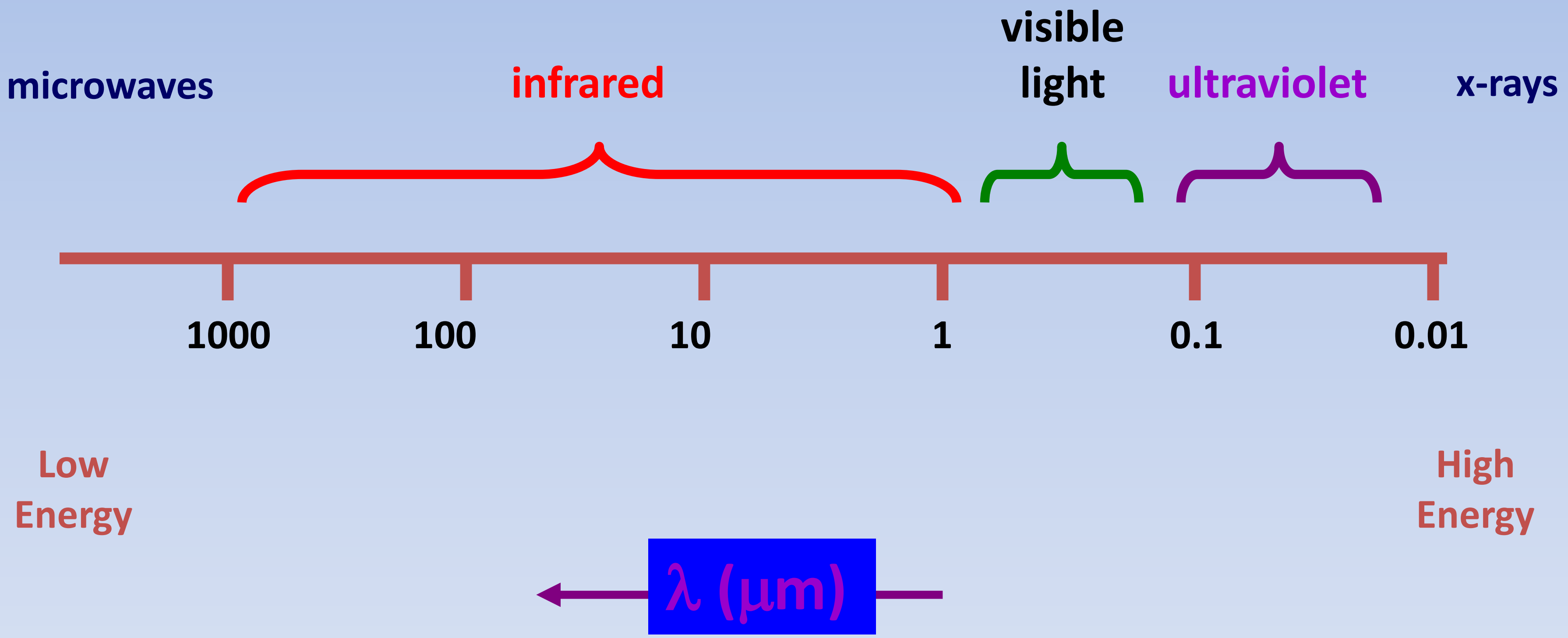
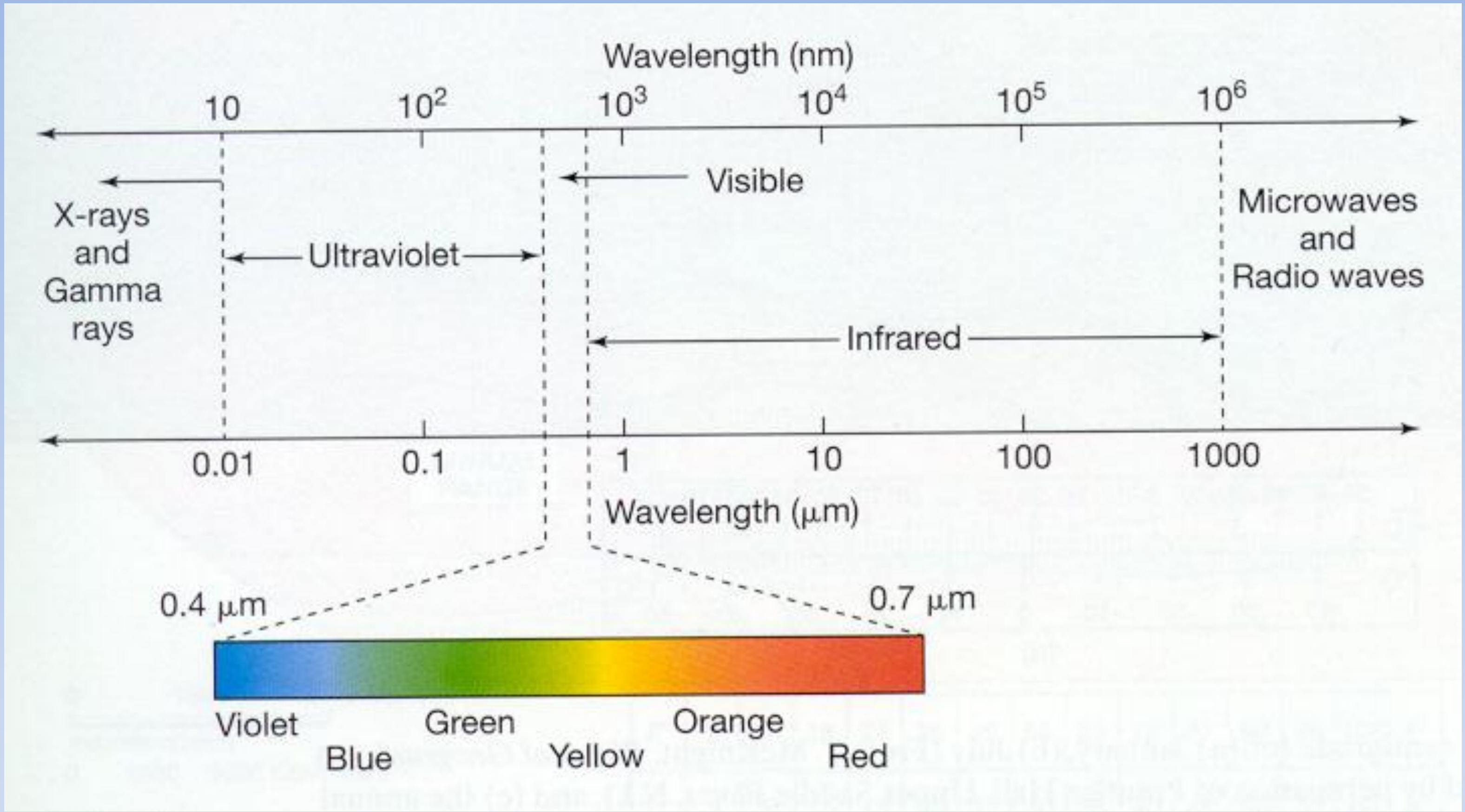
Electromagnetic Spectrum



	T (K)	λ_{max} (μm)	region in spectrum	F (W/m^2)
Sun	6000	0.5	Visible (yellow?)	
Earth	300	10	infrared	



- Blue light from the Sun is removed from the beam by Rayleigh scattering, so the Sun appears yellow when viewed from Earth's surface even though its radiation peaks in the green

	T (K)	**λ_{max} (μm)**	**region in spectrum**	**F (W/m^2)**
Sun	6000	0.5	Visible (green)	
Earth	300	10	infrared	

Thermal radiation

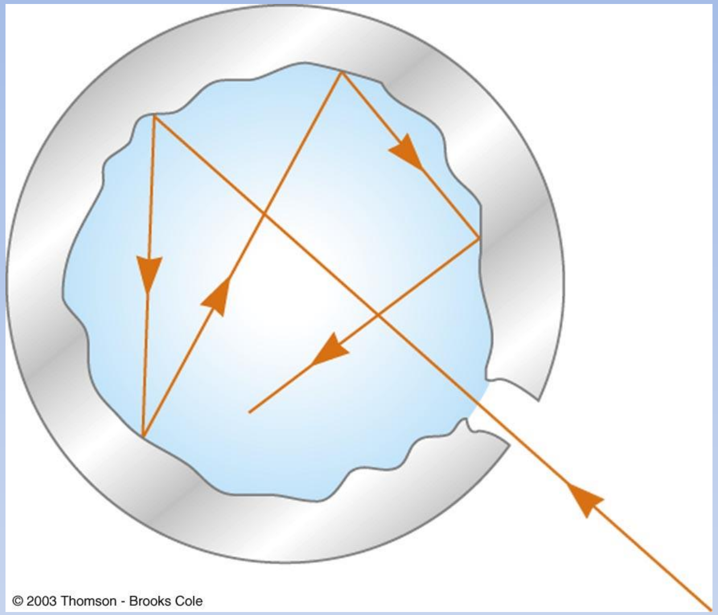
- Thermal radiation = e.m. radiation emitted by a body by virtue of its temperature
- Spectrum is continuous, comprising all wavelengths
- Thermal radiation formed inside body by random thermal motions of its atoms and molecules, repeatedly absorbed and re-emitted on its way to surface $\Rightarrow$ original character of radiation obliterated $\Rightarrow$ spectrum of radiation depends only on temperature, not on identity of object
- Amount of radiation actually emitted or absorbed depends on nature of surface
- Good absorbers are also good emitters (why??)

Black-body radiation

- "Black body"
  - perfect absorber
    - ideal body which absorbs all e.m. radiation that strikes it, any wavelength, any intensity
    - such a body would appear black $\Rightarrow$ **"black body"**
  - must also be perfect emitter
    - able to emit radiation of any wavelength at any intensity -- "black-body radiation"
- "Hollow cavity" ("Hohlraum") kept at constant T
    - hollow cavity with small hole in wall is good approximation to black body
    - thermal equilibrium inside, radiation can escape through hole, looks like black-body radiation

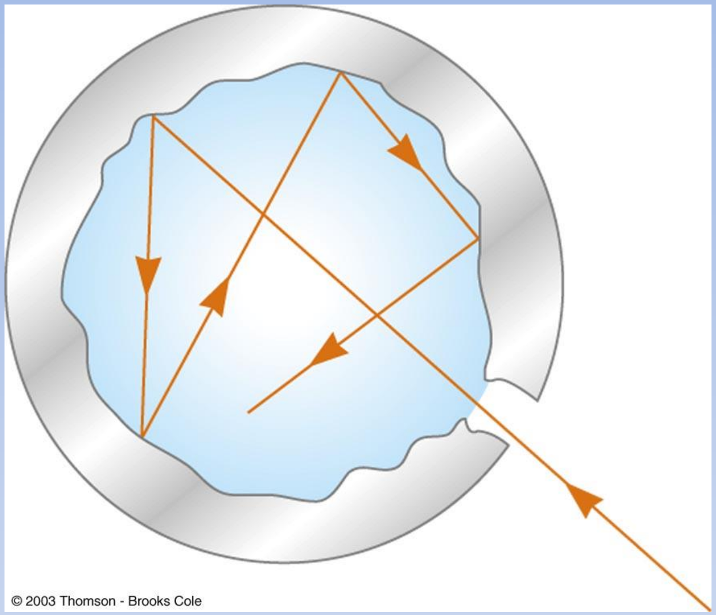
Studies of radiation from hollow cavity

- behavior of radiation within a heated cavity studied by many physicists, both theoretically and experimentally
- Experimental findings:
  - spectral density ρ(n,T) (= energy per unit volume per unit frequency) of the heated cavity depends on the frequency n of the emitted light and the temperature T of the cavity and nothing else.

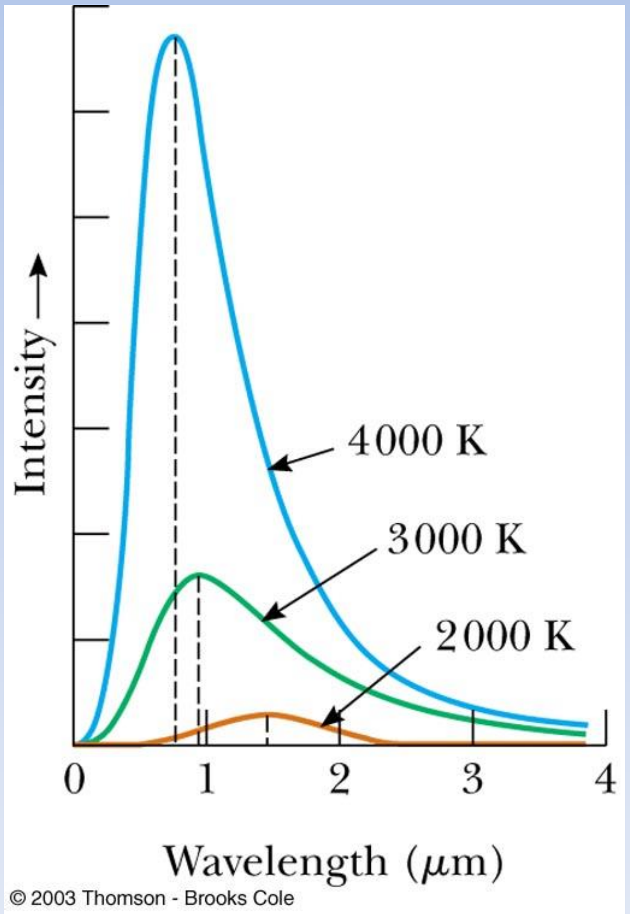


Black-body radiation spectrum

- Measurements of Lummer and Pringsheim (1900)



- Light radiated by an object characteristic of its *temperature*, not its surface color.
- Spectrum of radiation changes with temperature



various attempts at descriptions:

- Peak vs Temperature:
  $\lambda_{max} \cdot T = C$ (Wien's displacement law), $C = 2.898 \cdot 10^{-3}$ m K
- total emitted power (per unit emitting area)
  $P = \sigma \cdot T^4$ (Stefan-Boltzmann), $\sigma = 5.672 \cdot 10^{-8}$ W m^{-2} K^{-4}
- Wilhelm Wien (1896)
  $\rho(\nu,T) = a\,\nu^3\, e^{-b\nu/T}$, (**a** and **b** constants).
  OK for high frequency but fails for low frequencies.
- Rayleigh-Jeans Law (1900)
  $\boldsymbol{\rho}(\boldsymbol{\nu},T) = a\,\boldsymbol{\nu}^2\, T$ (**a** = constant).
  (constant found to be $= 8\pi k/c^3$ by James Jeans, in 1906)
  OK for low frequencies, but "ultra – violet catastrophe" at high frequencies

Planck's quantum hypothesis

- Max Planck (Oct 1900) found formula that reproduced the experimental results

$$\rho(\nu,T)=\frac{8\pi\nu^2}{c^3}E_{osc}$$

$$E_{osc}=\frac{h\nu}{e^{h\nu/kT}-1}$$

E_{osc} is the average energy of a cavity "oscillator"

- derivation from classical thermodynamics, but required assumption that oscillator energies can only take specific values $E = 0, h\nu, 2h\nu, 3h\nu, \ldots$ (using "Boltzmann factor" $W(E) = e^{-E/kT}$)

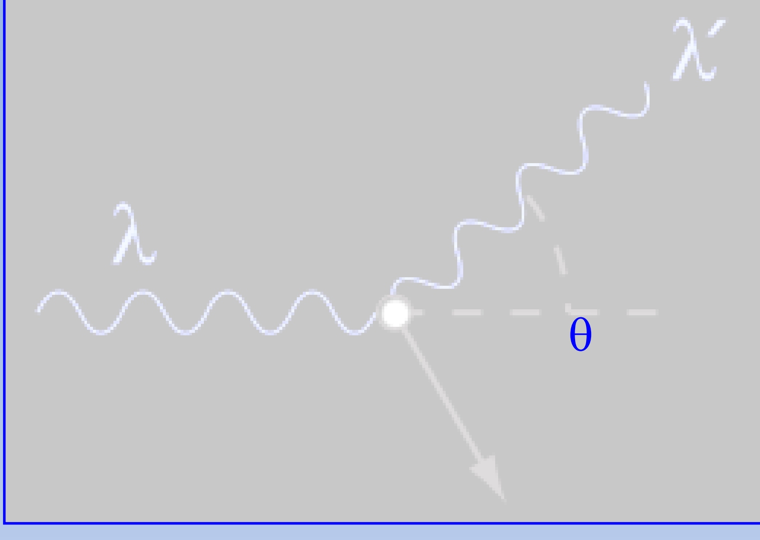
Consequences of Planck's hypothesis

- oscillator energies $E = nh\nu$, $n = 0,1,\ldots$;
  - $h = 6.626\ 10^{-34}$ Js = $4.13\ 10^{-15}$ eV·s
    now called Planck's constant
  - $\Rightarrow$ oscillator's energy can only change by discrete amounts, absorb or emit energy in small packets – "quanta";
    $E_{quantum} = h\nu$
  - average energy of oscillator
    $\langle E_{osc}\rangle = h\nu/(e^x - 1)$ with $x = h\nu/kT$;
    for low frequencies get classical result
    $\langle E_{osc}\rangle = kT$, $k = 1.38 \cdot 10^{-23}$ J·K^{-1}

Compton effect

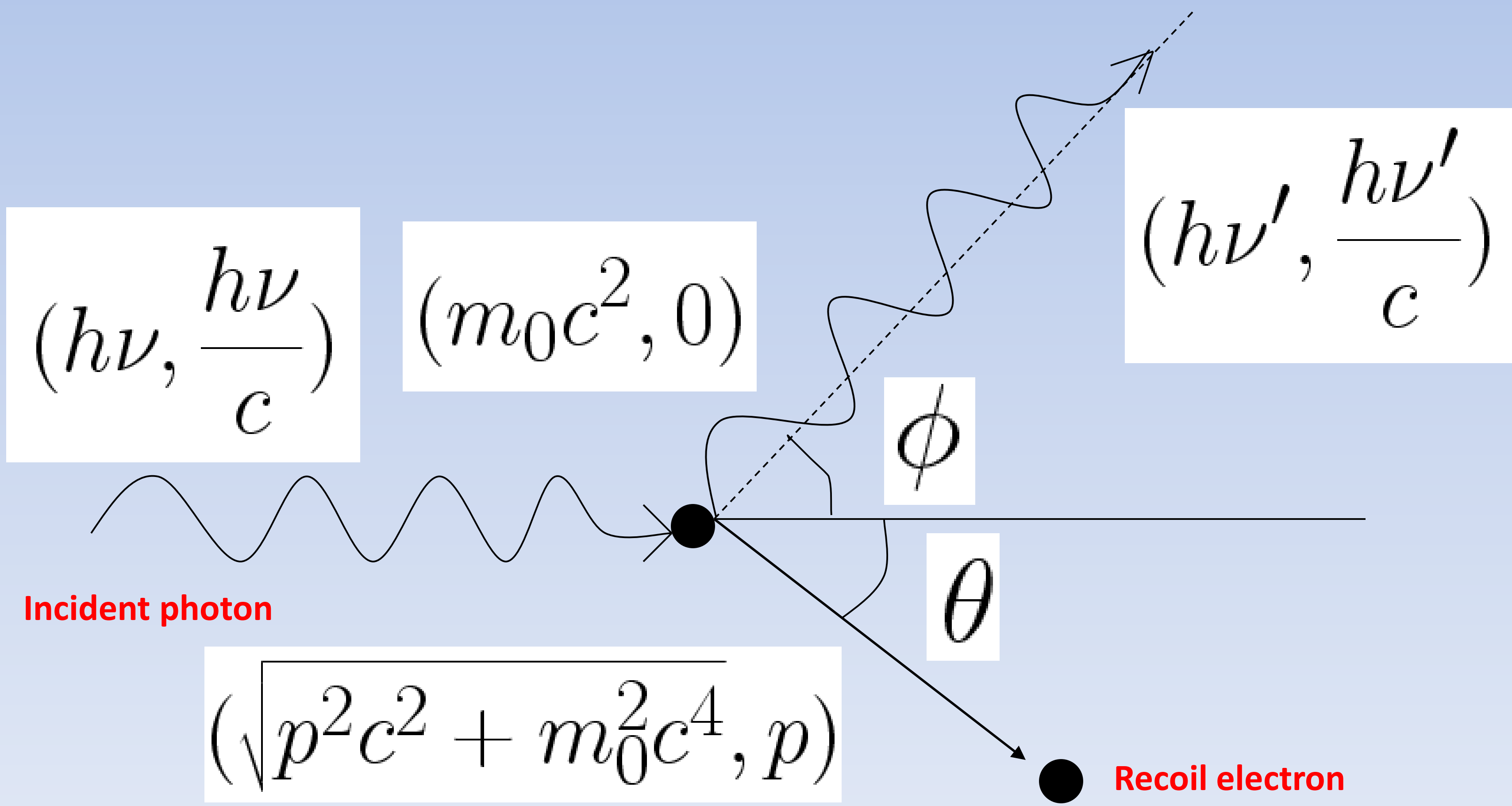
Compton Effect

$$\lambda - \lambda' = \frac{h}{m_e c^2}(1 - Cos\theta)$$



- What was the importance of Compton effect?
- Collision between two particles
  - Energy-momentum must both be conserved simultaneously
- Light consist of particles called photons
- What about phenomenon of Interference & diffraction?
- Logical tight rope of Feyman
- Light behaves sometimes as particles sometimes as waves

Partial transfer of photon energy



$$E^2 = p^2c^2 + m_0^2c^4$$

$$E = mc^2 = \frac{m_0}{\sqrt{1 - v^2/c^2}}c^2$$

$$KE_{classical} = \frac{1}{2}m_0v^2$$

$$h\nu - h\nu' = KE_{electron}$$

$$E_{photon} = pc$$

$$p_{photon} = \frac{h\nu}{c}$$

Conservation of momentum along initial photon direction

$$\frac{h\nu}{c} = \frac{h\nu'}{c}\cos\phi + p\cos\theta$$

Conservation of momentum along perpendicular to initial photon direction

$$\frac{h\nu'}{c}\sin\phi = p\sin\theta$$

$$p^2c^2 = (h\nu)^2 + (h\nu')^2 - 2(h\nu)(h\nu')\cos\phi$$

Energy conservation

$$h\nu + m_0c^2 = h\nu' + \sqrt{p^2c^2 + m_0^2c^4}$$

$$(h\nu)^2 + (h\nu')^2 + 2(h\nu)m_0c^2 - 2(h\nu)(h\nu')$$

$$-2(h\nu')m_0c^2 = p^2c^2$$

$$\frac{m_0 c}{h}\left(\frac{\nu}{c} - \frac{\nu'}{c}\right) = \frac{\nu}{c}\frac{\nu'}{c}(1 - \cos\phi)$$

$$\lambda' - \lambda = \frac{h}{m_0 c}(1 - \cos\phi)$$

Compton wavelength $\lambda_c = \frac{h}{m_0 c}$ ≅ 2.4 pm

Compton shift $\lambda' - \lambda = \lambda_c(1 - \cos\phi)$